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Modeling Social Welfare Functions Aligned with Income Taxation

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ABSTRACT

Objective: This study aims to develop and apply two advanced social welfare functions that enhance sensitivity to income inequality and risk aversion, focusing on improving the welfare of lower-income groups in the context of income tax policy design.

Methods: The study critiques classical social welfare models—such as utilitarian and aggregative approaches—for their limited responsiveness to inequality and social behaviors. It introduces two alternative frameworks: i) A function based on the Atkinson inequality index, capturing societal aversion to income disparities. ii) A function utilizing constant relative risk aversion (CRRA) utility, modeling individual welfare under income volatility. These models evaluate the effects of varying income tax rates on overall social welfare, accounting for average income and its distribution across societal strata.

Results: The analysis shows that increasing income tax rates across all income groups reduces social welfare due to lower post-tax incomes, even when redistribution is intended. Individuals with higher levels of risk aversion experience greater welfare losses, emphasizing the importance of incorporating inequality sensitivity and risk aversion into policy design.

Conclusion: The proposed social welfare functions offer more robust analytical tools for optimizing income tax policies. They promote equitable income distribution and improved social welfare by integrating distributive justice and risk-averse behavior. These models provide practical guidance for policymakers balancing economic efficiency with social justice.

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Introduction

Implementing efficient policies to reduce inequality has always been a challenge for governments. Systematic studies on inequality and the distribution of income and wealth from the perspective of development theories initially emphasized economic growth. However, the experiences of most developing economies, such as Latin American and African countries, and the severity of poverty in these regions led to a shift in focus after the 1980s toward the redistributive effects of policies and government transfer payments. In fact, without considering the distribution and allocation of taxes across different societal strata, social welfare cannot be enhanced, and this issue must be addressed by taking into account the factors affecting welfare (Salari-Esker et al., 2024).

Furthermore, addressing social justice is considered one of the fundamental necessities of societies today. Governments strive to achieve social justice and support vulnerable groups by designing and implementing programs and policies that reduce inequality and enhance overall social welfare. The principle of justice in distributing resources and opportunities is among the key concerns of policymakers, and legislation must be enacted to ensure equal access to facilities and opportunities. In this regard, modeling social justice and welfare, particularly through modern structural analysis methods, is crucial in effective policymaking (Badaghi et al., 2024).

Unequal allocations are not necessarily unjust. It is generally accepted that greater effort should be rewarded with greater benefits; larger polluters should bear a greater share of environmental costs; individuals with special needs or those who have been unfairly treated should be compensated; and so forth. Indeed, justice is one of the key concerns of individuals and policymakers (Zhiyong, 2024).

Poverty remains a critical global challenge that hinders sustainable development. Despite progress in reducing extreme poverty rates, millions still live below the international poverty line and face multidimensional deprivations, including limited access to health, education, and basic services (Zaki Bin Che Aat & Jaafar, 2025). Unequal income distribution increases class disparities, reduces work motivation, causes widespread poverty, political tensions, and pervasive corruption. Income inequality exists to some degree in all countries, with significant disparities between the incomes of the wealthy and the poor in developed and developing nations. However, the gap between the poor and wealthy classes is typically larger in less developed countries than in developed ones. This underscores the necessity of researching and analyzing the factors contributing to income inequality that lead to poverty. Poverty can be alleviated by investigating and identifying these factors, developing policy strategies and plans to reduce inequality, and implementing these policies, addressing many societal issues (Dehshiri et al., 2021). Taxes, as one of the fiscal policy tools available to governments, are a key factor affecting income distribution. In any economic system, taxation is considered one of its most critical components. The expansion

and diversification of economic activities on one hand, and the increasing role of governments on the other, in creating and expanding public services, social security, and government commitments in economic and social spheres, as well as efforts to achieve economic growth and equitable income distribution, have made tax collection and payment a significant and influential issue (Faramarzi et al., 2015).

Furthermore, social welfare modeling at managerial levels assists organizations in more accurately analyzing and predicting the effects of social and economic policies and programs. By simulating the complex interactions of various economic, social, and institutional factors, these models enable the optimization of resource allocation and the improved design of welfare programs. Specifically, managers can use these models to evaluate the impact of tax and transfer policies on the welfare of target populations and, based on the results, formulate more effective strategies to reduce inequality and enhance social justice (Rosania, 2024). Additionally, these modeling approaches significantly strengthen transparency, accountability, and justice in organizational decision-making processes, improving overall organizational performance (Shokrolehzadeh et al., 2024).

The impact of social welfare modeling on organizations can be examined from several dimensions. First, these models enable managers to predict policy decisions' economic and social consequences, reducing risks associated with inefficient planning. Second, through precise and scientific data analysis, organizations can enhance resource allocation in alignment with intelligent approaches, enabling more effective implementation of social justice and poverty reduction (Ghaderi & Mohammadi, 2024). Moreover, social welfare models foster improved intersectoral collaboration and strategic coordination in governmental organizations and institutions related to social policies, enhancing the development of human and organizational capacities (Shokrolehzadeh et al., 2024).

This study aims to address the following key questions: How can the social welfare function be modeled to reflect sensitivity to inequality and individuals' risk-averse behaviors under different income tax scenarios? Moreover, how can post-tax income data be incorporated into this welfare function? The necessity of this research is evident, as classical models often fail to address social behaviors and justice-oriented concerns adequately, and tax policies based on these models may, despite their aim of promoting justice, reduce social welfare. The innovation of this study lies in developing and presenting two social welfare functions based on the Atkinson index and a utility function with constant relative risk aversion, which can simultaneously assess the impacts of inequality and risk while assigning greater weight to lower-income groups. This enables a more precise analysis of the consequences of tax policies and provides a robust framework for improving policymakers' decision-making processes.

This research aims to model the determination of personal income tax in Iran. This modeling aims to enhance social welfare by considering satisfaction derived from post-tax income and reducing inequality (with attention to the income of others).

Literature Background

This section introduces the social welfare and utility functions developed by researchers in prior studies.

Theoretical Background

The purpose of this theoretical background section is to review and introduce social welfare functions that researchers have modeled in the past. This section is designed to identify previous formulas developed for analyzing social welfare, thereby strengthening the theoretical framework of the present study and providing a basis for comparing and developing new models. It should be noted that this section does not describe the operational model of the current study but rather explicates the formulas created in the past to explain social welfare, commonly referred to as classical models, which have served as the foundation for many studies.

Social welfare is a multifaceted state encompassing economic, social, and political dimensions, to preserve human dignity, fostering mutual responsibility among community members, and enhancing individual and collective capabilities. Social welfare refers to equitable access to essential services and facilities such as education, healthcare, housing, and employment opportunities, extending beyond material welfare to include psychological, social, and spiritual dimensions of individuals (Moftakheri et al., 2024). According to another definition, social welfare is a state of well-being in which individuals and communities can access adequate material resources (such as income and basic services), physical and mental health, and social and economic opportunities to lead a satisfying life. This concept encompasses economic and qualitative aspects such as security, social participation, and happiness (Adler, 2023).

Utilitarianism has been the most influential and dominant ethical theory for over a century. Initially, Pigou established the foundations of old welfare economics based on Bentham's moral philosophy in 1920, and subsequently, alongside public policies grounded in these ideas, it was presented in a more systematic form inspired by Jeremy Bentham. This approach focuses on the total utility derived by individuals from goods and services. Since total utility is composed of the utilities of individual members, if each individual pursues their personal satisfaction without interference, the maximum possible social welfare for the entire society will be achieved. Such a function in a society with no members is represented as follows:

$$W = \frac{\sum_{i=1}^{n} a_i (u_i)^{1-e}}{1-e} \quad (1)$$

Where W represents the social welfare function, Ui denotes the utility of the i-th individual, a_i is the weight assigned to each individual's utility, and e is the elasticity of substitution.

Furthermore, according to the properties of Benthamite utilitarian functions, which assume constant elasticity and assign equal importance to all individuals, the above function can also be expressed as follows:

$$w\,(U_1, U_2, \dots, U_n) = \sum_{i=1}^{n} U_i \quad (2)$$

In this function, a transition from one social state to another is justified only if it results in greater utility for the members of society.

Lerner and Pareto established the social welfare function based on ordinal utilities. This framework, known as Pareto efficiency, was elaborated by economists such as Kaldor, Hicks, Scitovsky, Samuelson, and Little, and became known as "new welfare economics." Since Pareto efficiency did not account for losses or damages and only considered changes in social welfare when the utility of no individual except one changes, the primary goal of its developers was to extend it to cases involving hypothetical compensation between winners and losers (Suzumura, 1987).

This approach fundamentally differed from old welfare economics in that it introduced ordinal utilities instead of cardinal ones. With this shift, although the concept of total utility or the sum of individual utilities was absent, the criterion for ranking social states remained dependent on the utilities derived from the personal domain of each individual. In fact, similar to Pigou’s approach, individual preferences within the domain of personal states were deemed sufficient for ranking social states in this framework. Consequently, in this approach, the social welfare function's origin lies in individuals' preferences and utilities within their personal states.

In the Bergson-Samuelson approach, the social welfare function is attributed to each individual in society, as this approach assumes that every individual possesses social preferences, and their ethical values regarding the distribution of social welfare are embedded within these preferences. In effect, the ranking of social states in this approach is derived from a combination of two types of weighting. The first is the weight that each individual assigns to their own utility, and the second is the weight that society assigns to the utility of each individual (Bergson, 1983).

The general mathematical form of this model is expressed as follows.

$$\Delta W = \left[\frac{dW}{dU^A}\right]\Delta U^A + \left[\frac{dW}{dU^B}\right]\Delta U^B \quad (3)$$

Where U^A andU^B represent the utilities of individuals A and B, respectively, and W denotes the social welfare of the society. The terms ΔU^A and the total number of individuals A and B changes due to policy implementation. On the other hand, in the second part of the ranking, the expressions $\frac{dW}{dU^A}$ and $\frac{dW}{dU^B}$ indicate how a one-unit change in an individual's utility affects social welfare and represent these two components in a society with n individuals as follows.

$$\frac{\partial W}{\partial \chi_i^n} = \frac{\partial W}{\partial U^n}\frac{\partial U^n}{\partial \chi_i^n} \quad (4)$$

The expression $\frac{\partial U^n}{\partial \chi_i^n}$ indicates how much the consumption of each unit of χ increases the utility of an individual, while the expression $\frac{\partial W}{\partial U^n}$ Shows how much an increase of one unit in an individual's utility contributes to social welfare (Fatemi Zardan et al., 2024).

Additionally, the social welfare function attributed to Pigou, as articulated by Bergson, incorporates distribution into its framework. According to this approach, any increase in national income that does not result in the transfer of income from the poor to the rich constitutes an improvement in economic welfare (Adler, 2024).

Bergson initially presents his proposed social welfare function as follows.

$$W = W(x_1, y_1, a_1^x, b_1^x, a_1^y, b_1^y, \dots, x_n, y_n, a_n^x, b_n^x, a_n^y, b_n^y, C^x, D^x, C^y, D^y) \quad (5)$$

Where X and Y are two types of consumer goods, C and D are factors of production other than labor, and a and b are two types of labor. Consumer goods are also a function of types of labor and other factors, as follows.

$$X = (A^x, B^x, C^x, D^x) \quad (6)$$

$$Y = (A^y, B^y, C^y, D^y) \quad (7)$$

If we consider production quantities other than labor constant, then the social welfare function will be as follows:

$$W = W(x_1, y_1, a_1^x, b_1^x, a_1^y, b_1^y, \dots, x_n, y_n, a_n^x, b_n^x, a_n^y, b_n^y) \quad (8)$$

For the sake of uniformity of discussion and ease of reaching her desired conclusion, Bergson considers the aforementioned social welfare function as the following separable form.

$$W = W\{S^1(x_1, y_1, a_1^x, b_1^x, a_1^y, b_1^y), \dots, S^n(x_n, y_n, a_n^x, b_n^x, a_n^y, b_n^y)\} \quad (9)$$

The functions discussed thus far pertain to utilitarianism. Now, other social welfare functions will be examined.

John Rawls, in 2000, argues that external factors should not influence competition. Accordingly, he considers individual talents and natural abilities, which are involuntary, as opportunities akin to religion, race, or social class. He does not regard them as inherently entitling individuals to advantages (Alizadeh & Zolghadr, 2022).

In this theory, while Rawls accepts two principles—freedom in competition and equality of opportunity—his interpretation of opportunities differs. He includes factors such as individual talents and natural abilities as part of opportunities that should be distributed equally. From his perspective, distributions should be implemented to benefit the least advantaged groups in terms of initial opportunities, thereby compensating for their disadvantages.

Rawls' theory of justice implies that rational individuals, in an original position, agree on a form of income distribution under a veil of ignorance, where participants in the decision-making process are unaware of their eventual position in the final income distribution. The veil of ignorance among individuals (taxpayers) leads them to support redistributive policies favoring the most deprived groups, as they do not know which income class they will belong to and may assume they could be among the least advantaged. Consequently, they view such redistribution as aligning with their personal interests.

Based on the difference principle, the social welfare function attributed to Rawls is expressed as follows (Varian, 2014).

$$W(u_1, \dots, u_n) = min\{u_1, \dots, u_n\} \quad (10)$$

Where $u_1, \dots, u_n$ represent the utilities of n individuals, and W denotes the social welfare of the society.

The perspectives of Amartya Sen are also worth examining. The space that Sen considers for ranking social states is the space of capabilities, which refers to the ability to choose a type of life that each values. Each individual deems specific actions or states valuable, ranging from basic matters such as adequate nutrition and the ability to avoid preventable diseases to more complex aspects such as the capacity to participate in social life and maintain self-respect. An individual's capability refers to the various combinations of these achievable and feasible functionings, and the

"capability set" comprises the different vectors of functionings from which an individual can choose.

Consequently, in Amartya Sen's social welfare function, each individual's utility is derived from the opportunities and capabilities that enable the flourishing of their life. Accordingly, the capability space ensures that individuals' utilities are incorporated into the social welfare function solely within the domain of capabilities, and the ranking of social states is determined within this space.

The general form of the social welfare function is expressed as follows:

$$W = W(S, \theta) \tag{11}$$

Where S represents the average income of the society, and θ denotes the income inequality within the society. Additionally, these two are functions of x, where x represents the income of each individual. The constraints and conditions of this function are expressed as follows:

$$\frac{\partial W}{\partial S} > 0 \quad ; \quad \frac{\partial W}{\partial \theta} < 0 \tag{12}$$

In descriptive terms, social welfare increases with an increase in the average income of society and decreases with an increase in income inequality within society. In fact, an increase in an individual's income affects social welfare in two ways: first, by increasing the total and average income of society, and second, by altering the level of equality or inequality in society. Consequently, if an increase in an individual's income reduces inequality, social welfare will increase. However, if inequality increases, social welfare will only increase if the effect of the increase in the average income of society outweighs the effect of the increased inequality.

$$\left|\frac{\partial W}{\partial \theta}\frac{\partial \theta}{\partial x_i}\right| < \frac{\partial W}{\partial S}\frac{\partial S}{\partial x_i} \tag{13}$$

Empirical Background

In this section, internal (Persian) studies are first discussed, followed by international studies, organized chronologically by publication year.

Internal Studies

The article titled Comparing the Effects of Social Welfare on Economic Growth in Iran and Selected Developing Countries examines the impact of social welfare indicators on economic growth in Iran and several developing countries from 2000 to 2018. This study employs a panel data model and the Generalized Method of Moments (GMM) for data analysis, using the Osberg

composite social welfare index as a welfare measure. This index encompasses four main dimensions: consumption, wealth, income distribution, and economic security. The results indicate that an increase in the social welfare index has a positive and significant effect on economic growth in the studied countries, including Iran, with improvements in income distribution and economic security playing a key role in fostering economic growth. Through comparative analysis, the study evaluates Iran's position among developing countries, highlighting that weaknesses in economic security and income inequality are significant barriers to improving economic growth in Iran (Moftakheri et al., 2024).

The article by Alizadeh et al. (2022) focuses on identifying key factors affecting social welfare in Iran's economy using a Bayesian averaging approach. The study adopts the Theil social welfare index as a welfare measure and analyzes time-series data from 1996 to 2018. By estimating 260,000 regressions, eight key variables were identified: exchange rate, misery index, tax revenues, oil revenues (with adverse effects), urbanization growth rate, economic openness, health indices, and information technology (with positive effects). The findings suggest that equitable economic policies and reducing income inequality can enhance social welfare (Alizadeh et al., 2022).

The article Utility and Social Welfare Function in Iranian Provinces (Investigating the Process of Changes and Convergence of Welfare) aims to examine trends in changes and convergence of welfare across Iranian provinces, utilizing a logarithmic utility function and Amartya Sen's social welfare function. The logarithmic utility function was chosen due to its ability to reflect diminishing marginal utility from income increases and its computational simplicity. Sen's social welfare function was selected because it is sensitive to average income and income distribution, playing a significant role in assessing social welfare. The results show that during the study period, Iranian provinces exhibited an improving trend in social welfare. However, complete convergence in welfare levels across provinces was not observed, with significant disparities persisting. These disparities are primarily attributed to unequal income distribution and differences in human development indices across provinces (Fatemi Zardan et al., 2021).

The article Marginal Utility Elasticity of Social Welfare Function Welfare Weights of Provinces in Iran investigates the impact of tax policies on social welfare in Iranian provinces by modeling personal income tax using an agent-based simulation and artificial intelligence approach. It employs an explicit social welfare function and an implicit utility function derived from the agent-based model. The use of agent-based simulation accounts for adaptive and learning behaviors of economic agents in response to tax policies and enables large-scale testing of economic policies. This model learns tax policies based on observable data without prior assumptions about individual behavior (Abdoli & Shirdel, 2010).

The article Optimal Effective Income Tax Rate in Iran's Economy: An Application of Rawlsian Social Welfare seeks to determine Iran's optimal income tax rate using a Rawlsian social

welfare approach. It employs a Rawlsian social welfare function to maximize the welfare of the least advantaged individuals. While a specific utility function is not mentioned in the summary, it can be assumed that individuals' utility is implicitly incorporated into the Rawlsian social welfare model, linked to their income. The results indicate that the optimal income tax rate is one that, while generating government revenue, maximizes benefits for low-income individuals and reduces inequality, as improving the condition of the poorest is a priority in Rawls' framework (Mehrabani & Nassiri-Aghdam, 2013).

The article Social Welfare Function Based on Social Preferences of Individuals in an Islamic Society develops and analyzes a social welfare function that models individual preferences within the framework of Islamic values and principles. Critiquing conventional social welfare functions, such as utility-based and Bernoulli-Nash functions, this study proposes a function that considers not only individual utility but also social justice, equitable resource distribution, and Islamic ethical values. Thus, the social welfare function aims to maximize total utility and emphasize maintaining balance and justice in welfare distribution, ensuring improvements in the welfare of less advantaged groups. This function, rooted in individuals' social preferences in an Islamic society, combines individualism and collectivism, where welfare maximization must align with justice and human dignity. Unlike purely economic ones, the study demonstrates that such a function provides a more comprehensive criterion for assessing social welfare in Islamic societies and guides welfare policies toward more equitable resource distribution. Additionally, with its sensitivity to inequality and adherence to religious values, this social welfare function illuminates the path to achieving optimal welfare in Islamic societies, emphasizing that social welfare must simultaneously ensure economic growth, social justice, and ethical considerations (Karami Esfeh, 2013).

The article Social Welfare Changes in Iran (Pareto and Non-Pareto Approaches of the Cardinal Social Welfare Function) analyzes changes in social welfare in Iran using the cardinal Bergson-Samuelson social welfare function. It employs both Pareto (focusing on improving the condition of better-off individuals) and non-Pareto (examining overall welfare changes without regard to distribution) approaches. The findings indicate that social welfare in Iran has generally followed an upward trend but with fluctuations, and less advantaged provinces are moving toward convergence with the national welfare average. This article highlights the importance of considering welfare distribution and using cardinal functions for more precise social welfare analysis and regional policymaking (Ebadi et al., 2013).

The article Introduction of Two-Dimensional Utility in the Social Welfare Function from the Perspective of the Islamic Value Approach examines whether two-dimensional utility (encompassing positive and negative aspects of individual satisfaction) alone can serve as an appropriate criterion for defining a social welfare function within an Islamic framework. The study concludes that a social welfare function in an Islamic approach must consider not only individual

welfare but also justice, human dignity, and Islamic ethical values. Therefore, an Islamic social welfare function should go beyond purely economic criteria and two-dimensional utility, reflecting an Islamic society's social preferences and values (Jaberi & Sabbaqi, 2019).

Hosseini, in the article Income Distribution in Iran Using Gini and Atkinson Indices from 2001 to 2013, employs the Gini and Atkinson indices to analyze income distribution. Hosseini argues that the Gini coefficient is more sensitive to income distribution in middle-income groups and cannot distinguish between distribution patterns when Lorenz curves intersect. Consequently, the Gini and Atkinson indices complement each other (Hosseini, 2015).

The article Investigating Amartya Sen's Social Welfare Function in Iran: A Theoretical and Empirical Analysis examines social welfare in Iran using Amartya Sen's social welfare function. Designed based on axiomatic theories and combining average income and its distribution, this function considers welfare levels and income inequality, providing a more comprehensive measure of social welfare. The findings emphasize that Sen's social welfare function, due to its consideration of income distribution and justice, is an effective tool for analyzing social welfare in Iran and can guide economic and social policymaking to improve welfare and reduce inequality (Sayyadzadeh & Ahmadi, 2006).

Heydari and Khodadadkashi, in the article Investigating Income Distribution in Iran Using Theil, Atkinson, and Gini Indices, measure income inequality in urban and rural areas and across household socioeconomic characteristics using raw data from the Household Expenditure and Income Survey and the Theil, Atkinson, and Gini indices. The findings indicate that household characteristics influence income distribution, and all three indices effectively model equality (Heydari & Khodadadkashi, 2008).

International Studies

The article Top Income Taxation: Efficiency, Social Welfare, and the Laffer Curve by Lundberg (2024) examines the optimization of high-income tax rates in advanced economies. Using the framework of a social welfare function and the concept of the welfare possibility frontier, which analyzes the trade-off between tax revenue and economic efficiency, the author demonstrates that current tax rates in many countries are below the optimal level that could increase tax revenue without reducing social welfare. Based on general equilibrium analysis and social weighting of individuals' welfare, the model concludes that the optimal tax rate in some countries could be 10 to 12 percent higher than the current rate. This article precisely analyzes the interplay between economic efficiency and social justice within the framework of social welfare functions (Lundberg, 2024).

The article A Note on Welfarist Versus Non-Welfarist Social Welfare Function, examines the differences between welfarist and non-welfarist social welfare functions. The authors present a

simple model demonstrating how a non-welfarist social welfare function can be transformed into or aligned with a welfarist one. This study highlights the importance of selecting an appropriate social welfare function in economic analysis and policymaking, contributing to a better understanding of social welfare concepts (An, 2024).

In the article " Social Welfare Functions and Health Policy: A New Approach, " Adler explores the framework of social welfare functions in the context of health policies. The authors investigate how government policy outcomes can be converted into comparable welfare values across individuals, introducing concepts such as the "social value of risk reduction." The models used include prioritized welfare functions that assign greater weight to less advantaged individuals, applicable in health policy analysis (Adler, 2024).

The article Designing a Model for Realizing Social Justice Based on Article 78 of the Sixth Development Plan with Structural Equation Modeling develops a model for achieving social justice within Article 78 of Iran's Sixth Development Plan. This study employs a mixed-method (qualitative and quantitative) approach. In the qualitative phase, 20 organizing themes were identified through thematic analysis and semi-structured interviews with 11 experts (selected via snowball sampling) and categorized into a thematic network. In the quantitative phase, structural equation modeling and confirmatory factor analysis validated the impact of the identified factors. The results indicate that achieving the social justice model, accountability, responsibility, and meritocracy in distributive justice are the top three priorities. While no explicit mathematical formula for a social welfare function is provided, the study uses social justice to evaluate supportive policies and reduce social harms. The methodology includes analyzing data collected from 200 randomly selected members of the Ministry of Welfare using statistical software for structural equation modeling (Zaki Bin Che Aat & Jaafar, 2024).

In Application of Pigou's Social Welfare Function in Bergson's Framework for Health Policy Design, Adler develops a novel framework for analyzing the effects of health policies on social welfare. Focusing on weighing the utility of low-income groups, the study employs theoretical analysis and policy simulations to examine healthcare resource allocation. The results show that equitable policies can reduce health inequalities and enhance social welfare, emphasizing the importance of integrating distributive justice considerations into health policymaking (Adler, 2024).

Additionally, Adler (2023), in Measuring Social Welfare: An Introduction, examines concepts and methods for measuring social welfare, providing a comprehensive framework for modeling social welfare functions. The study reviews the Bergson-Samuelson and Pigou social welfare functions, highlighting their limitations in interpersonal utility comparisons and introducing modern approaches focusing on inequality and distributive justice. The methodology includes theoretical analysis and case studies, underscoring the importance of addressing structural

inequalities in economic policymaking and emphasizing the need to integrate justice and equality considerations into welfare analyses (Adler, 2023).

Berg and Piacquadio (2022), in their study on the role of tax expenditures in welfare policies, analyze their impact on income distribution and social welfare globally. The authors use international data to demonstrate that such expenditures can significantly reduce inequality and enhance social welfare, but their design and targeting are critical. The models combine micro- and macroeconomic analyses, assisting policymakers in evaluating the real impacts of taxes and expenditures on welfare (Berg & Piacquadio, 2022).

The article The AI Economist: Improving Equality and Productivity with AI-Driven Tax Policies by Zheng et al. (2020) presents an innovative approach to designing tax policies using model-free reinforcement learning (RL). This method trains a social planner and economic agents within an AI-based economic simulation. Taxes are calculated by applying different rates to income brackets, with tax rates determined through a deep neural network. This approach offers high flexibility, as any social objective (e.g., increasing equality or productivity) can be defined as a reward function without requiring prior global knowledge. Experiments demonstrate that tax policies trained in simulations are consistent with economic intuition and perform well in real-world settings with human participants. The article proves that AI-based economic simulators can be powerful tools for designing optimal tax policies, balancing social justice and productivity (Zheng et al., 2020).

The article Measuring Social Welfare Gains in Social Assistance Programs: An Application to European Countries by Barcena-Martin and Ayala (2019) analyzes the increase in social welfare resulting from social assistance programs in European countries. The study employs a social welfare function that considers income, income distribution, and inequality effects to more accurately measure welfare improvements. Specifically, it examines how social assistance programs reduce poverty and inequality, enhancing social welfare. Using a social welfare function that accounts for inequality provides a more precise and comprehensive assessment of these programs' benefits, showing that merely increasing income is insufficient; improving income distribution and social justice is also critical. This approach helps policymakers better understand the real impacts of welfare programs and design more targeted policies to enhance social welfare (Barcena-Martin & Ayala, 2019).

The article Assessment of Inter-Regional Convergence in Social Welfare Based on the Sen Function: Russian Case Study by Malkina (2017) examines the trend of social welfare convergence across different regions in Russia. The study uses Sen's social welfare function, which combines per capita income and an inequality index (Gini coefficient), thus considering welfare levels and distribution. The results indicate a trend of social welfare convergence across regions in Russia, meaning that welfare disparities between regions have gradually decreased (Malkina, 2017).

Additionally, Emmanuel Saez, along with Thomas Piketty, in the article Optimal Taxation of Top Labor Incomes: A Tale of Three Elasticities (2011), builds on Mirrlees' framework to argue for a tax system that seeks broader distributive justice. The study analyzes the optimal income tax problem for top labor incomes by considering three behavioral response channels to taxation: (1) the standard supply-side channel through reduced effort, (2) the tax avoidance channel by shifting income to minimize taxes, and (3) the compensation bargaining channel through efforts to influence wage-setting. The authors derive simple optimal tax rate formulas as functions of three elasticities corresponding to these channels, showing that the models produce significantly different predictions for the optimal marginal tax rate (Saez & Piketty, 2011)

Materials and Methods

Modeling the Social Welfare Function

At this stage, the aim is to develop two social welfare functions based on the theoretical foundations of social welfare functions and the models constructed thus far, which can serve as appropriate guidelines for determining income tax policies.

As discussed in previous sections, foundational theories have not adequately addressed the issues of equality or justice, particularly concerning prioritizing the poor, which represents a primary weakness of many earlier models. Equality relevant to increasing social welfare involves individuals comparing themselves to others. In reality, it cannot be assumed that an increase in an individual's income necessarily enhances societal welfare. For instance, if an individual's income increases by 10% while the income of others in society increases by 50%, that individual may experience dissatisfaction, reducing their utility. If the group experiencing a 10% income increase is sufficiently large, the collective dissatisfaction may outweigh the utility gained by the group with a 50% income increase. Consequently, societal utility may decrease despite overall income growth in such an income distribution scenario. Therefore, attention to equality and the weighting of individuals' utility plays a critical role and must be considered. Additionally, consideration of individuals' risk aversion is also essential.

Social Welfare Function Using the Atkinson Concept

The first concept utilized for modeling the social welfare function is the Atkinson index. Atkinson (1970) developed an equality index for income distribution. He recognized that value judgments implicitly influence inequality measures, and these judgments should be explicitly incorporated into inequality indices. The value judgments reflected in the social welfare function determine the degree of society's aversion to inequality and, therefore, must be integrated into the inequality index. In essence, formulating the inequality index should explicitly indicate the cost society will bear to reduce inequality. Atkinson emphasizes the relationship between efficiency and equality,

arguing that an inequality index should be designed to allow policymakers to determine how much income or current production society is willing to sacrifice to achieve a specific reduction in inequality. In other words, the social welfare function specifies the rate at which society is prepared to forgo a portion of its production and income to achieve a defined reduction in inequality.

Welfare-based inequality measures are most effectively used when ranking through various forms of distributional dominance cannot provide a definitive ranking. However, it is worth noting that welfare-based measures can be applied in any case where welfare analysis is needed. Nevertheless, these measures generally have less analytical power than distributional dominance methods.

In fact, distributional dominance provides a partial ranking, as there are cases where the welfare of two income distributions cannot be ranked. Distributional dominance is also an ordinal ranking, meaning it only indicates a preference for one distribution over another without specifying the magnitude of that preference.

When distributional dominance loses effectiveness, or there is interest in composite numerical measures representing the entire income distribution, welfare-based measures may provide a complete ranking among alternative income distributions. However, this comes at the cost of more restrictive assumptions regarding how social welfare is represented.

The main conceptual differences between distributional dominance and welfare-based inequality measures are as follows.

Table 1. Main Conceptual Differences Between Distributional Dominance and Welfare-Based Inequality Measures

Distributional Dominance	Type	Welfare-Based Measures
Partial ranking	Type of Ranking	Complete ranking
Ordinal ranking	Characteristics	Single numbers
Broad categories of social welfare functions	Assumptions about the Social Welfare Function	Precise determination of the social welfare function
Weak, requires validation with other social welfare functions	Robustness of Results	Strong

(Adler, 2023)

Welfare-based measures can provide a complete ranking by converting income distributions into a single number. As such, they enable a cardinal ranking that relies on precisely determining the form of the social welfare function. Consequently, the robustness of the results derived from these indices is weak, necessitating validation and comparison with other social welfare functions.

In contrast, distributional dominance is characterized by partial ranking and is considered an ordinal ranking as it does not require the precise specification of the social welfare function. Additionally, because the assumptions regarding the social welfare function are minimal, the robustness of its results is strong.

Now, the use of welfare-based measures is discussed. The Atkinson index of inequality plays a prominent role in welfare-based measures. The Atkinson index is directly related to the class of additive social welfare functions:

$$W = \frac{1}{N}\sum_{i=1}^{n} U(y_i) \tag{14}$$

Equation (14) indicates that social welfare is represented by average utility. According to Atkinson, the form of the utility function U is expressed as follows:

$$\begin{aligned} U(y_i) &= \frac{1}{1-\varepsilon} y_i^{1-\varepsilon} \qquad \varepsilon \neq 1 \\ U(y_i) &= log y_i \qquad\qquad \varepsilon = 1 \end{aligned} \tag{15}$$

Where the parameter ε represents inequality aversion. According to Equation (15), if there is no aversion to inequality, the utility becomes equivalent to the individual's average income. In this case, a higher average income leads to an increase in social welfare.

This function has a positive first-order derivative for the social welfare function:

$$\frac{\partial W}{\partial y_i} = \frac{1}{n}\frac{1-\varepsilon}{1-\varepsilon} y_i^{1-\varepsilon-1} = \frac{y^{-\varepsilon}}{n} > 0 \tag{16}$$

This implies that as aversion to inequality increases, greater weight is assigned to lower incomes. The negative second-order derivative indicates this ($-\varepsilon\frac{y^{-\varepsilon-1}}{n} < 0$) and the concavity of the graph.

Since ε reflects a value judgment, the precise determination of Equation (14) depends on the value of ε.

The Atkinson inequality index's cornerstone is the Equally Distributed Equivalent Income (EDE) concept. EDE income is the level of income that, if received equally by every individual in the income distribution, would enable society to achieve the same level of social welfare as that obtained with actual incomes (Bellu & Liberati, 2005).

Figure 1 illustrates the concept of EDE. This diagram depicts the social welfare function in the space of individual incomes.

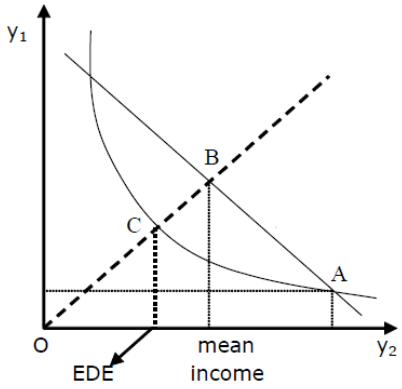

Figure 1. Diagram of Equally Distributed Equivalent Income

The vertical axis represents the income of individual 1, and the horizontal axis represents the income of individual 2. Suppose the income distribution is such that point A is an equilibrium where y2>y1. The utilitarian social welfare function (a straight line) prevails without inequality aversion. With this social welfare function, the only way to have equal incomes at the same level of welfare is to allocate the average income to both individuals (point B). Since inequality aversion is zero, there is no incentive to reduce income to achieve societal equality.

With inequality aversion, a convex social welfare function prevails. Starting from point A, we can find a point where incomes are distributed equally at the same welfare level. Because the social welfare function is convex, point C must be less than the average income. Point C lies on the 45-degree line and has the same social welfare as point A. Although total income (the sum of the two individuals' incomes) is less than at A, this is compensated for by increased equality in the distribution. This is because, given positive inequality aversion, society is now willing to accept a lower share of income to achieve better equality (Blaut & Liberati, 2006).

Graphically, the EDE income equals the income level corresponding to point C. Equality is measured by the ratio OC/OB. When each individual has the same income level, or when the social welfare function is utilitarian (no inequality), this ratio equals 1; thus, the Atkinson inequality index is expressed as follows:

$$A(\varepsilon) = 1 - \frac{OC}{OB} = 1 - \frac{y_{EDE} * \sqrt{2}}{\bar{y} * \sqrt{2}} = 1 - \frac{y_{EDE}}{\bar{y}} \tag{17}$$

Intuitively, this index indicates the proportion of income society is willing to forgo to achieve equal incomes. To operationalize the Atkinson inequality index, an expression for the equally distributed equivalent (EDE) income is required, which is derived using equation (15):

$$U(y_{EDE}) = \frac{1}{1-\varepsilon} y_{EDE}^{1-\varepsilon} \tag{18}$$

According to Figure 1, social welfare in equation 14 must be equal to equations 15 and 17, that is:

$$SWF = \frac{1}{n}\sum \frac{y_i^{1-\varepsilon}}{1-\varepsilon} = \frac{1}{n} n \frac{y_{EDE}^{1-\varepsilon}}{1-\varepsilon} = \frac{y_{EDE}^{1-\varepsilon}}{1-\varepsilon} \quad (19)$$

Therefore, given any income distribution, the Equally Distributed Equivalent (EDE) income can be easily calculated for different levels of inequality aversion. Different levels of inequality aversion yield different values of y_{EDE}. For zero inequality aversion, the equally distributed equivalent income is simply the average income level, and as inequality aversion increases, y_{EDE} decreases. From the above relation, the formula for y_{EDE} can also be derived:

$$y_{EDE} = \left[\frac{1}{n}\sum y_i^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \quad (20)$$

Social welfare function using the concept of constant relative risk aversion

Kenneth Arrow and John Pratt independently utilized the concept known as Constant Relative Risk Aversion (CRRA) in financial and economic discussions.

In his works (1965), Kenneth Arrow scientifically and mathematically introduced and analyzed the concept of relative risk aversion. He defined relative risk aversion as a measure that indicates how an individual's tendency to avoid risk changes with changes in their wealth level. Arrow modeled the utility function so that the coefficient of relative risk aversion remains constant, meaning the ratio of risk aversion to the individual's wealth does not change. According to the explanations and standard definitions provided by Arrow and John Pratt in their papers, the formula for relative risk aversion is defined as follows (Pratt, 1964):

$$R(W) = \frac{U''(W).W}{U'(W)} \quad (21)$$

Next, if the coefficient R(W) is independent of W (i.e., constant), the utility function takes the form of the CRRA (Constant Relative Risk Aversion) utility function, whose standard form is given as follows[1]:

$$U(c_i) = \frac{c_i^{1-\rho} - 1}{1-\rho} \quad for\ \rho \neq 1 \ , \ c_i = y_i(1-t) \quad (22)$$

$$U(c_i) = ln(c_i) \quad for\ \rho = 1 \quad (23)$$

[1] Here, for the purposes of this article, instead of using w*w*, y*y* is used, which represents after-tax income.

The utility function $U(c_i)$ depends on after-tax income, where c_i represents after-tax income, ρ is the coefficient of risk aversion, y_i is the pre-tax income, and t is the tax rate. Higher values of ρ indicate greater risk aversion; if ρ is zero, it implies risk neutrality. The CRRA function is also used to describe how individuals make decisions when facing uncertainty or risk. It helps to understand how satisfied a person is with their after-tax income, given their attitude toward risk.

In fact, if the coefficient of relative risk aversion R(W) is independent of wealth level (i.e., constant), then the utility function U(W) must be of a specific form that satisfies this condition. This condition is equivalent to solving the following differential equation:

$$U'(W)\frac{\rho}{W} + U''(W) = 0 \quad (24)$$

Solving this differential equation leads us to equations 21 and 22, which correspond to the CRRA utility function.

Moreover, to assign weights proportional to the concept of justice mentioned in the Atkinson section, inspired by Atkinson's concept, the following relation is introduced for the weight of each utility:

$$\omega(c_i) = \left(\frac{c_i}{\bar{c}}\right)^{-\varepsilon} \quad (25)$$

By assigning greater weight to individuals with lower incomes (through a higher ε), this model prioritizes policies that benefit those with lower incomes, reflecting a stronger preference for equality.

As a result, the social welfare function is expressed as follows:

$$SWF = \sum_{i=1}^{N} \omega(c_i)U(c_i) \quad (26)$$

Results

Using individual income data from the Statistical Center, individuals are classified into five income groups (from low-income to high-income). Based on risk aversion and inequality criteria, social welfare conditions under different tax regimes have been calculated for each group.

In fact, referring to income data from the Statistical Center and due to the similarity in responses to inequality and risk between adjacent deciles, group 1 corresponds to the first and second deciles, group 2 corresponds to the third and fourth deciles, and group 3 corresponds to the ninth and tenth deciles of society.

Various studies have investigated the parameters of inequality aversion and risk aversion, generally using ordinal numbers based on income classification. In this article, the ordinal numbers for these two variables are adopted based on medians reported by Fredrik Carlsson and colleagues in 2001 in the study titled "Are People Averse to Inequality or Only Risk Averse?" which used the Gini coefficient and coefficient of variation. Additionally, a questionnaire assessing risk aversion and inequality aversion was completed by 25,000 respondents and organized by Ada Frio and Javier Ramos in 2010.

Risk aversion and inequality aversion for each group are as follows:

Table 2. Coefficients of risk aversion and inequality aversion for income groups

Income Group	Inequality Aversion (ε)	Risk Aversion (ρ)
1	1.5	2.5
2	1.2	2.0
3	0.9	1.5
4	0.6	1.0
5	0.3	0.5

Specifically, individuals with lower incomes exhibit greater aversion to inequality and risk, and as an individual's income increases, their tolerance for risk increases. In comparison, their aversion to others' higher incomes decreases. Consequently, the parameters epsilon and rho are specified for the income groups as in Table 2.

Furthermore, assuming higher taxes for higher income groups (with an assumed 5% tax difference compared to the previous group), seventeen taxation scenarios have been defined:

Table 3. Different Tax Percentage Scenarios

State	Groups 1	Group 2	Group 3	Group 4	Group 5
1	0%	5%	10%	15%	20%
2	5%	10%	15%	20%	25%
3	10%	15%	20%	25%	30%
4	15%	20%	25%	30%	35%
5	20%	25%	30%	35%	40%
6	25%	30%	35%	40%	45%
7	30%	35%	40%	45%	50%
8	35%	40%	45%	50%	55%
9	40%	45%	50%	55%	60%
10	45%	50%	55%	60%	65%
11	50%	55%	60%	65%	70%
12	55%	60%	65%	70%	75%
13	60%	65%	70%	75%	80%
14	65%	70%	75%	80%	85%
15	70%	75%	80%	85%	90%
16	75%	80%	85%	90%	95%
17	80%	85%	90%	95%	100%

With different tax rates, risk aversion, and inequality classification numbers, as well as income statistics of individuals in society, the amount of social welfare for both selected formulas is as follows:

Table 4. Social welfare

State	CRRA Welfare	Atkinson Welfare
1	116,412,904	280,270
2	113,008,381	267,915
3	109,496,208	255,310
4	105,865,357	242,432
5	102,102,756	229,251
6	98,192,700	215,736
7	94,116,045	201,845
8	89,849,009	187,530
9	85,361,408	172,727
10	80,613,865	157,356
11	75,553,197	141,307
12	70,104,206	124,430
13	64,153,485	106,498
14	57,512,815	87,148
15	49,815,514	65,712
16	40,071,968	40,599
17	(43,735,452)	408

Unit: index

The results can also be displayed in a graph, as shown in Figure 2.

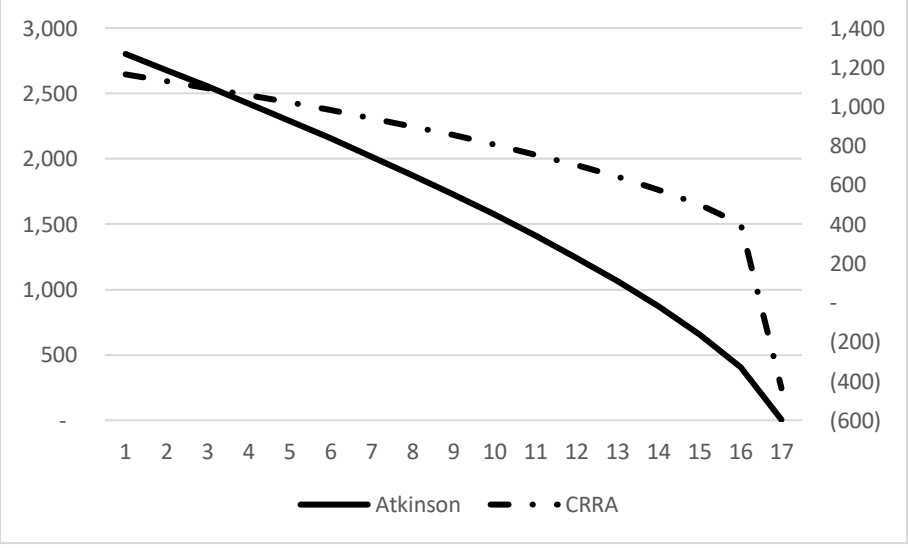

Figure 2. Social welfare diagram in different tax situations

Also, social welfare changes are as follows.

Table 5. Changes in social welfare

Scenario	Welfare Changes Based on Constant Relative Risk Aversion Model	Welfare Changes Based on the Atkinson Model
1	—	—
2	-2.90%	-4.41%
3	-3.15%	-4.70%
4	-3.30%	-5.04%
5	-3.55%	-5.43%
6	-3.82%	-5.88%
7	-4.15%	-6.44%
8	-4.53%	-7.10%
9	-5.00%	-7.89%
10	-5.56%	-8.90%
11	-6.25%	-10.18%
12	-7.21%	-11.94%
13	-8.49%	-14.41%
14	-10.35%	-18.14%
15	-13.38%	-24.60%
16	-19.56%	-38.27%
17	-209%	-99.00%

The findings of this research indicate that classical models based on the simple aggregation of individual utilities, due to insufficient attention to social behaviors, aversion to inequality, and sensitivity to the conditions of low-income groups, fail to accurately reflect social realities and justice-oriented concerns. In contrast, the proposed models based on the Atkinson index and the utility function with constant relative risk aversion (CRRA) have been able to simultaneously incorporate society's sensitivity to inequality and risk-averse behavior in policy analyses.

In this study, two social welfare functions have been modeled, both designed to allocate greater weight to the welfare of low-income groups. This feature makes the reduction in post-tax income of these groups have a greater impact on the reduction of overall social welfare. In other words, these welfare functions more precisely demonstrate how income inequality and income reduction among low-income individuals can affect the total welfare of society.

Statistical analyses and simulations in this research, which utilized income data of 12,000 individuals divided into five income classes based on income deciles and applied the model to 85 different tax scenarios, show that increasing income tax rates for all income groups, even if aimed at income redistribution and inequality reduction, leads to a reduction in social welfare. This welfare decline results from decreased post-tax income, which reduces consumption and individual welfare.

Moreover, the CRRA-based model shows that individuals with higher degrees of risk aversion are more sensitive to income fluctuations caused by tax changes and experience greater welfare reductions. This finding highlights the importance of considering individuals' financial and psychological behaviors in designing tax policies. Specifically, in the CRRA model, the percentage changes in welfare vary, with sharp decreases in the final scenarios (especially scenarios 15 to 17), indicating high sensitivity of individuals with high risk aversion (low-income groups with ρ up to 3.0) to income fluctuations from heavy taxation. In contrast, the Atkinson model, with welfare changes from -4.41% in scenario 2 to -99% in scenario 17, shows a sharper decline due to greater weighting on inequality aversion (ε up to 2.0 for low-income groups), consistent with the paper's emphasis on sensitivity to low-income groups. Overall, as tax rates increase (particularly from scenario 10 onward), the decline in welfare accelerates.

Additionally, analysis of the percentage welfare changes across 17 taxation scenarios for the CRRA and Atkinson welfare models, using quantitative measures like correlation or disparity ratios, reveals that both models consistently show welfare reduction with increased tax rates, confirmed by a high correlation coefficient of over 0.98, indicating similar directional patterns. The average relative difference between the changes in the two models is about 28.5%, rising to 111% in the final scenarios due to severe drops. Similarities include sensitivity to low-income groups and similar responses to increased taxes, while differences lie in the severity of decline; the Atkinson model, emphasizing inequality, exhibits a sharper decline (up to 99%), while the CRRA model, focusing on risk aversion, responds more moderately until scenario 17, reflecting differing priorities in these models regarding social welfare aspects.

In conclusion, the findings stress that tax policies must be carefully designed with attention to income distribution and the sensitivity of low-income groups to maintain social justice while preventing an overall decline in societal welfare. Utilizing social welfare functions based on the Atkinson index and constant relative risk aversion provides powerful tools for analyzing and optimizing tax policies, aiding policymakers in making better decisions.

Validation of the Method

For validation of the model, the internal validity method was applied. Internal validity is a process used in the validation of scientific models to ensure that the internal structure of the model is consistent with economic theories and expected logic. This method tests the model's response to controlled changes in key variables. The main goal is to check whether the hypothetical relationships within the model operate correctly.

To examine internal validity, the parameters of inequality aversion (ε) and risk aversion (ρ) were varied across different ranges to evaluate the response of the Atkinson and CRRA models to these changes. Using hypothetical initial income data (10, 20, 40, 70, 100 units) and a fixed tax

rate of 10 percent for all classes (scenario 1), the baseline values of ε (2.0, 1.5, 1.0, 0.5, 0.2) and ρ (3.0, 2.5, 2.0, 1.5, 1.0) were first applied, which resulted in Atkinson welfare of 15.3 and CRRA welfare of 4.5. Then, ε was sequentially changed to 0.5, 1.0, 1.5, 2.0, and 2.5 for all classes, and ρ was sequentially changed to 1.0, 1.5, 2.0, 2.5, and 3.0. For the Atkinson model, as ε increased from 0.5 to 2.5, welfare decreased from 18.2 to 12.7, indicating greater sensitivity to inequality and weighting of low-income classes. For the CRRA model, as ρ increased from 1.0 to 3.0, welfare decreased from 5.1 to 3.9, reflecting the risk-averse behavior of lower-income individuals.

These changes are consistent with economic logic; increasing ε in the Atkinson model reduces welfare because inequality is perceived as more severe, and increasing ρ in the CRRA model lowers welfare because more risk-averse individuals view income fluctuations more negatively. Calculations were done using Python code for greater accuracy.

In fact, the internal validity of the models is confirmed because parameter changes align with theoretical expectations. However, the sharp decrease in welfare at ε above 2.0 or ρ above 3.0 (e.g., 12.7 and 3.9) may indicate limitations in the applicable domain, especially under excessive inequality or risk conditions. This suggests that in policymaking, ε and ρ should be kept within reasonable ranges (e.g., 0.2 to 2.0 for ε and 1.0 to 3.0 for ρ) to avoid unrealistic results.

Conclusion

The present study, aiming to model a social welfare function aligned with income taxation and focusing on concepts of inequality and risk aversion, concludes that the use of classical utility-based models, due to their neglect of social behaviors and sensitivity to the conditions of low-income groups, cannot accurately represent the economic and social realities of society. This paper has derived two social welfare functions based on the Atkinson index and the utility function with constant relative risk aversion.

The two proposed models (social welfare based on the Atkinson index and utility function with constant relative risk aversion) share similarities with previous empirical studies such as "Utility and Social Welfare Functions in Iranian Provinces" (Fatemi Zardan et al., 1400), which used the logarithmic function and Amartya Sen's index to analyze welfare changes and addressed incomplete convergence among provinces—both showing attention to income distribution and inequality sensitivity. Similarities are also seen in "Optimal Effective Income Tax Rate in Iran" (Mehrbani and Nasiri Aghdam, 1392), emphasizing Rawlsian welfare, which focuses on low-income groups and distributive justice priorities. However, the proposed models combine the Atkinson index and constant relative risk aversion to provide a more comprehensive approach that simultaneously considers inequality (via ε) and risk-averse behavior (via ρ), unlike classic models such as Pigou or Samuelson, which rely solely on summing individual utilities and pay less attention to social aspects.

The distinguishing feature of the proposed models lies in their ability to assign greater weight to low-income groups and sensitivity to income fluctuations, depths not seen in empirical works like "Social Welfare Changes in Iran" (Ebadi et al., 1391) or "Marginal Utility Elasticity of Social Welfare Functions" (Abdoli and Shirdel, 1389). By using parameters for inequality and risk aversion (based on medians from Carlson et al. (2001) and Freier-e-Carbonel and Ramos (2010)), these models offer a more data-driven approach compared to theoretical predecessors like Rawls or Amartya Sen, who focus more on philosophical principles or capabilities. On the other hand, the common ground includes emphasizing welfare decline with increasing taxation and the importance of fair income distribution, which aligns with prior empirical findings; notably, tax rate increases also lead to welfare reduction, though the proposed models exhibit greater decline due to more sensitive parameters. These differences and similarities indicate that the proposed models compensate for classical model deficiencies and integrate empirical and theoretical aspects to provide a more advanced tool for tax policy-making that can help optimize social welfare and reduce inequality, as underscored in the article's conclusion.

The findings indicate that models based on the Atkinson equality index and constant relative risk aversion are suitable analytical tools for evaluating tax policies as they simultaneously consider society's sensitivity to inequality and individuals' risk-averse behavior, assigning more weight to the welfare of low-income groups. According to the results, increasing income tax rates, even for redistribution and inequality reduction purposes, ultimately reduces social welfare. This decline mainly results from decreased disposable income, reducing consumption, and individual welfare. Therefore, tax policies should be designed to achieve social justice and reduce inequality without dampening economic incentives or overall societal welfare.

Moreover, findings show that individuals with higher risk aversion are more sensitive to income fluctuations from tax policies and experience greater welfare reductions. This highlights the importance of considering behavioral and psychological traits in tax system design and suggests that uniform tax policies across income groups will not necessarily yield social justice.

The distinct performances of the two models under specific conditions are notable: the Atkinson model, with greater weight on low-income groups (ε up to 2.0), outperforms in scenarios of severe inequality (e.g., higher taxes on upper classes), reflecting welfare reduction up to 99% and better representing distributive justice. Conversely, the CRRA model, focusing on risk aversion (ρ up to 3.0), performs better under high income volatility by modeling risk-averse behaviors more accurately. This distinction is evident in welfare percentage changes data, indicating that Atkinson is more suitable for redistributive policies while CRRA fits risk impact analysis. Unlike earlier, often one-dimensional models, these two combine data-driven and social behavior sensitivity, providing a more sophisticated tool for optimizing tax policies. This is consistent with the paper's emphasis on enhancing overall welfare and reducing inequality.

Overall, this research emphasizes that to improve social welfare and achieve fairer income distribution, policymakers should adopt advanced social welfare models that simultaneously consider inequality and risk aversion. This approach can lead to more effective and realistic tax policy designs, ultimately contributing to developmental goals and increased public satisfaction.

It is suggested that subsequent studies use these two models to optimize tax rates in diverse tax scenarios, including varying rates and targeted taxation on lower-income individuals, through utility maximization approaches.

Social welfare variables are relatively incorporated in the proposed models, but their completeness depends on data and model assumptions. The Atkinson model, incorporating inequality aversion and income distribution weighting, covers variables like income inequality and sensitivity to low-income conditions consistent with classical indices like the Gini or Atkinson index. The CRRA model, with its risk aversion parameter, accounts for risk-averse behavior and income volatility consistent with economic concepts like Pratt's utility function (1964). However, both models insufficiently address longer-term tax effects on labor incentives, investment, or mental health. Moreover, a lack of dynamic data on social preferences or externalities may limit the analysis. Therefore, while capturing key variables such as income, inequality, and risk, future behavioral and long-term variables integration is necessary for more comprehensive modeling.

Data Availability Statement

Data are available on request from the authors.

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Conflict of interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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